



## TD 3 — The Two-Phase Method

### Lecture 4 · Exercise sheet

#### Exercise 1 Preparing the equations

- (a)  $\max z = 2x_1 + 3x_2$  s.t.  $x_1 + x_2 \leq 8$ ,  $x_1 - x_2 \geq 2$ ,  $x_2 \geq 1$ ,  $x_1, x_2 \geq 0$ .
- (b)  $\min z = x_1 + 2x_2 + x_3$  s.t.  $x_1 + x_2 + x_3 = 6$ ,  $x_1 - 2x_2 \leq -2$ ,  $x_3 \leq 4$ ,  $x_1, x_2, x_3 \geq 0$ .
- For each problem, prepare the equations with Rule 4.1. Which variables are basic at the start, and what are their values?
  - Write the Phase 1 objective  $W$  and the first tableau of Phase 1, with the  $W$ -line cleaned (Rule 4.2).
  - Solve problem (a) completely with the two-phase method.
  - Problem (a) has two variables: draw its feasible set and check your answer. Show on the drawing the corner reached at the end of Phase 1.

#### Exercise 2 Charging a battery (TD 1, Exercise 3)

Recall:  $t_F$  and  $t_S$  are the hours on the fast and on the slow charger.

$$\min z = 300t_F + 80t_S \quad \text{s.t.} \quad 2t_F + t_S \geq 24 \text{ (energy, kWh)}, \quad t_F + t_S \leq 16 \text{ (time, hours)}, \quad t_F, t_S \geq 0.$$

- Prepare the equations and write the first tableau of Phase 1.
- Run Phase 1. Which corner do you reach? Is it a good plan?
- Run Phase 2 and give the cheapest plan. Check it with a drawing of the feasible set.
- The last  $z$ -line has 220 under  $e_1$  and 140 under  $s_2$ . Guess the new cost if the battery needs 25 kWh, then if 17 hours are available. Check both guesses by solving the two binding constraints.

#### Exercise 3 Exactly all the assembly hours

The drone workshop must use exactly its 40 assembly hours (Lecture 2, special situation C):

$$\max z = 40x_1 + 60x_2 \quad \text{s.t.} \quad x_1 + 2x_2 = 40, \quad 3x_1 + 2x_2 \leq 60, \quad x_2 \leq 18, \quad x_1, x_2 \geq 0.$$

- Prepare the equations. How many artificial variables do you need?
- Run Phase 1. Which corner of the drone workshop do you reach?
- Run Phase 2 and give the best plan. Compare with Lecture 2.

#### Exercise 4 A problem with no feasible plan

$$\max z = x_1 \quad \text{s.t.} \quad x_1 + x_2 \geq 5, \quad 2x_1 + 3x_2 \leq 6, \quad x_1, x_2 \geq 0.$$

1. Run Phase 1 and conclude with Rule 4.3.
2. Give the value of the artificial variable at the end of Phase 1, and explain what it means.

### Exercise 5 Degeneracy

$$\max z = 3x_1 + 2x_2 \quad \text{s.t.} \quad x_1 + x_2 \leq 4, \quad x_1 \leq 2, \quad 2x_1 + x_2 \leq 6, \quad x_1, x_2 \geq 0.$$

1. Run the simplex method with the usual rules. Where is there a tie in the ratio test?
2. In the last tableau, which basic variable is equal to 0? Draw the feasible set and explain this with the lines that pass through the optimal corner.
3. Go back to the tie, and choose the *other* leaving variable. Continue the simplex method. Show that you make a degenerate pivot, and that you reach the same point with the same value of  $z$ .
4. Show that the constraint  $2x_1 + x_2 \leq 6$  is redundant.

### Exercise 6 To go further (at home)

Recall example (b) of Lecture 1, whose standard form is Exercise 7 of TD 1:

$$\min 3x_1 + 2x_2 \quad \text{s.t.} \quad x_1 + x_2 \geq 2, \quad x_1 - 2x_2 \leq 4, \quad x_1 \geq 0, \quad x_2 \text{ free.}$$

1. Starting from the standard form (with  $x_2 = x_2^+ - x_2^-$ ), add an artificial variable and solve the problem with the two-phase method.
2. Give the best plan in the original variables  $(x_1, x_2)$  and the minimum cost. Compare with Lecture 1.
3. *Big-M*. Write the Big-M objective of this problem and its first objective line after cleaning (you do not need to solve it).