



TD 2 — From the Picture to the Simplex Method

Lectures 2 and 3 · Exercise sheet

Exercise 1 Control boards

Recall the model of TD 1, Exercise 2: x_1, x_2 = numbers of Basic and Pro boards produced per day (profit in thousand DA),

$$\max z = 12x_1 + 20x_2 \quad \text{s.t.} \quad \begin{cases} 2x_1 + 4x_2 \leq 480 & (\text{assembly machine, minutes}) \\ 3x_1 + 2x_2 \leq 360 & (\text{test bench, minutes}) \\ x_2 \leq 100 & (\text{special sensors}) \\ x_1, x_2 \geq 0. \end{cases}$$

A. With a picture

1. Draw the feasible set and compute the coordinates of all its corners.
2. Solve the problem with the graphical method, and check with the corner method. Give the optimal plan, the optimal value, the binding constraints, and the number of special sensors not used.

B. With the simplex method

3. Write the first dictionary, then run the simplex method with dictionaries (Method 3.1). At each step write: who enters, the ratio test, who leaves, the new dictionary, and the new corner with its value of z .
4. Explain with the last z -line why the last corner is optimal. On your figure of question 1, draw the path followed by the simplex method.

Exercise 2 Corners and basic solutions

$$\max z = 3x_1 + 2x_2 \quad \text{s.t.} \quad x_1 + x_2 \leq 4, \quad x_1 + 3x_2 \leq 9, \quad x_1 \leq 3, \quad x_1, x_2 \geq 0.$$

A. With a picture

1. Draw the feasible set, compute its corners, and find the optimal plan with the corner method.

B. With the simplex method

2. Write the standard form with slack variables s_1, s_2, s_3 . At each corner of question 1, give the values of the five variables. How many of them are zero? (These are the basic feasible solutions, Key fact 3.1.)
3. Solve the problem with the simplex method in tableaux (Method 3.2). Which corners are visited? Draw the path on your figure, and compare the result with question 1.

Exercise 3 Two special cases, seen twice

- (a) $\max z = x_1 + x_2$ s.t. $-x_1 + x_2 \leq 2, x_1 - 2x_2 \leq 2, x_1, x_2 \geq 0$.
- (b) $\max z = 2x_1 + 4x_2$ s.t. $x_1 + 2x_2 \leq 6, 2x_1 + x_2 \leq 6, x_1, x_2 \geq 0$.

A. With a picture

- For each problem, draw the feasible set and say which case it is: unique optimum, infinitely many optima, unbounded problem, or infeasible problem. Justify your answer.

B. With the simplex method

- Solve (a) with tableaux. Which signal of Lecture 3 appears? Give a family of feasible plans $x(t), t \geq 0$, along which $z \rightarrow +\infty$, and compare with your picture.
- Solve (b) with tableaux. Which signal of Lecture 3 appears? Do one more pivot to find a second optimal corner, give all the optimal plans, and compare with your picture.

Exercise 4 At home — The drone workshop, by another path

Recall the drone workshop of the lectures: x_1, x_2 = numbers of drones A and B made per week (profit in thousand DA),

$$\max z = 40x_1 + 60x_2 \quad \text{s.t.} \quad \begin{cases} x_1 + 2x_2 \leq 40 & (\text{assembly hours}) \\ 3x_1 + 2x_2 \leq 60 & (\text{testing hours}) \\ x_2 \leq 18 & (\text{cameras}) \\ x_1, x_2 \geq 0. \end{cases}$$

A. With a picture

- Draw the feasible set, compute its corners O, A, B, C, D, and find the optimal plan with the corner method.

B. With the simplex method

- Solve the problem with tableaux, but choose x_1 as the first entering variable (the lecture chose x_2). Which corners are visited? How many pivots are needed? Draw the path on your figure and compare with the path of the lecture.