Answers to Exercise 1:

For two consecutive experiments, the total number of possible outcomes is calculated using the addition principle:

$$\text{Total outcomes} = \sum_{i=1}^{m} n_i$$

where m is the number of outcomes for the first experiment and n_i is the number of outcomes for the second experiment corresponding to each outcome of the first.

Answers to Exercise 2:

To find the possible initials consisting of 2 letters:

$$\text{Total initials} = 26 \times 26.$$

To guarantee at least two students share the same initials, at least $26^2 + 1$ students are needed in a class.

Answers to Exercise 3:

For the 10 volumes of an encyclopedia arranged randomly:

- Total arrangements without restrictions:
$$10!.$$
- If volumes 7, 6, and 5 must be placed next to each other in that specific order, treat them as a single unit:
$$8!.$$

Answers to Exercise 4:

1. Total 3-letter words with no restrictions:
$$26^3.$$
2. Total with at least 2 consonants:
$$\text{Total} = (\text{3 consonants}) \text{ or } (\text{2 consonants, 1 vowel}).$$
$$21^3 + 3 \cdot 21^2 \cdot 5$$
3. Exactly 2 consonants and 1 vowel:
$$21^2 \cdot 5 \cdot 3,$$
select 2 consonants, 1 vowel, and arrange them (ccv or cvc or vcc).
4. At most 1 vowel:
$$\text{Total} = \text{no vowel or } (\text{2 consonants, 1 vowel}) = (\text{3 consonants}) \text{ or } (\text{2 consonants, 1 vowel}).$$
$$21^3 + 3 \cdot 21^2 \cdot 5$$
Same as question 2.

Answers to Exercise 5:

1. Total ways to visit 7 family members on the same day:
$$7!.$$
2. If 4 are visited on Eid and 3 the next day:
$$(7)_4 \cdot 3! = 7!.$$

Answers to Exercise 6:

1. Total football teams from 22 people:
$$\binom{22}{11}.$$
2. Teams formed with 3 goalkeepers including at least 1:
$$\text{Choose 1 goalkeeper and 10 from 19 others} \Rightarrow 3 \cdot \binom{19}{10}.$$

Answers to Exercise 7:

A company with 2 managers, 3 administrators, and 11 workers:

1. Total ways without restrictions:
$$\binom{16}{4}.$$
2. 1 manager, 2 administrators, and 1 worker:
$$\binom{2}{1} \cdot \binom{3}{2} \cdot \binom{11}{1}.$$
3. Workers and administrators cannot be together:
   - Solution 1: Choose them from ($admins \cup managers$) or ($workers \cup managers$):
   $$\binom{3+2}{4} + \binom{11+2}{4} = \binom{5}{4} + \binom{13}{4}$$
   - Solution 2: Consider all the cases:
   $$\left.\begin{array}{rc} 4W: & \binom{11}{4} \\ 3W,1M: & \binom{11}{3}\binom{2}{1} \\ 2W,2M: & \binom{11}{2}\binom{2}{2} \\ 3A,1M: & \binom{3}{3}\binom{2}{1} \\ 2A,2M: & \binom{3}{2}\binom{2}{2} \end{array}\right\} \overset{A.P.}{\Longrightarrow} Total = \binom{11}{4} + \binom{11}{3}\binom{2}{1} + \binom{11}{2}\binom{2}{2} + \binom{3}{3}\binom{2}{1} + \binom{3}{2}\binom{2}{2}$$
   - Solution 3: We consider all the total number - the number where admins and workers meet (3W,1A),(2W,2A),(2W,1A,1M),(1W,3A),(1W,2A,M),(1W,1A,2M):
   $$\binom{16}{4} - \left(\binom{2}{2}\binom{3}{1}\binom{11}{1} + \binom{2}{1}\binom{3}{2}\binom{11}{1} + \binom{3}{3}\binom{11}{1} + \binom{2}{1}\binom{3}{1}\binom{11}{2} + \binom{3}{2}\binom{11}{2} + \binom{3}{1}\binom{11}{3}\right).$$

Answers to Exercise 8:

To divide a class of 100 students into groups of sizes 15, 18, 20, 21, and 26, we can use the binomial coefficient approach:

$$\text{Total choices} = \binom{100}{15} \times \binom{85}{18} \times \binom{67}{20} \times \binom{47}{21}$$

The last group will consist of the remaining 26 students, which is represented as $\binom{26}{26} = 1$.

Thus, the total number of choices for this division is given by:

$$\text{Total choices} = \binom{100}{15} \times \binom{85}{18} \times \binom{67}{20} \times \binom{47}{21} = \frac{100!}{15! \cdot 18! \cdot 20! \cdot 21! \cdot 26!}$$

Answers to Exercise 9:

In a committee of 10 people, a chairperson, a secretary and a treasurer should be selected

1. Knowing that accumulation is not allowed, in how many ways can these offices be allocated if
   i. There are no restrictions?
      Answer: $(10)_3 = \binom{10}{3}3! = 720$
   ii. A and B cannot be together?

Answer 1: $\bar{A}\bar{B} \vee \bar{A}B \vee A\bar{B}$

$$\binom{8}{3}3! + \binom{1}{1}\binom{8}{2}3! + \binom{1}{1}\binom{8}{2}3!$$

Answer 2: $\bar{A} \vee A\bar{B}$

$$\binom{9}{3}3! + \binom{1}{1}\binom{8}{2}3!$$

Answer 3: $Total - AB$

$$\binom{10}{3}3! - \binom{2}{2}\binom{8}{1}3!$$

   iii. C and D are together or not at all?
      Answer: $CD + \bar{C}\bar{D}$:

$$\binom{2}{2}\binom{8}{1}3! + \binom{8}{3}3! = 384$$

   iv. F must have a task?
      Answer:

$$\binom{2}{2}\binom{8}{1}3! + \binom{8}{3}3! = 216$$

   v. F only accepts the task of chairperson?
      Answer: F president or not all

$$\binom{9}{2}2! + \binom{9}{3}3!$$

2. Knowing that accumulation is allowed, in how many ways can these offices be allocated if there are no restrictions?
   Answer:

$$10^3$$

Answers to Exercise 10:

- If the student has to answer 5 out of 8 questions:

$$\binom{8}{5} = 56$$

- If the student has to answer at least 3 out of the first 5 questions: So he exactly 3 or 4 or 5 out of the first 5 questions and 2 from the remaining 3:

$$\binom{5}{3} \times \binom{3}{2} + \binom{5}{4} \times \binom{3}{1} + \binom{5}{5} = 46$$

Answers to Exercise 11:

- For the word "ALGER":

$$5! = 120$$

- For the word "ANNABA":

$$\frac{6!}{3! \times 2!} = 60$$

- For the word "TIZIOUZOU":

$$\frac{9!}{(2!)^4}$$

- For the word "MISSISSIPPI":

$$\frac{11!}{4! \times 4! \times (2!)} = 34,650$$

Answers to Exercise 12:

In how many ways can 8 people be seated?

- In a row,
  1. **If there are no restrictions:** The total arrangements of 8 people is given by:

  $$8!$$

  2. **Two persons A and B stay together:** Treat A and B as a single unit. This creates 7 units (AB and 6 others):

  $$7! \times 2!.$$

  3. **There are 4 adults and 4 children, each adult has only children as neighbors and vice versa:** Arrange adults and children alternatively:

  $$\text{Arrangement: } ACACACAC \text{ or } CACACACA.$$

  Total arrangements:

  $$4! \times 4! \times 2.$$

  4. **The number of adults is 6, they must stay together:** Treat the 6 adults as a single unit. This creates 3 units (ADULTS and 2 children):

  $$3! \times 6!.$$

  5. **There are 4 children, each accompanied by his mother who stays next to him:** Treat each mother-child pair as a single unit. This creates 4 units (2! permutations of each pair):

  $$4! \cdot (2!)^4.$$

- Around a table
  1. **If there are no restrictions:**

  $$7!$$

  3. **There are 4 adults and 4 children, each adult has only children as neighbors and vice versa:**

  $$3! \times 4!$$

Answers to Exercise 13:

Group the athletes by nationality:

- Algerian block (4 athletes)
- Libyan block (3 athletes)
- Tunisian block (2 athletes)

$$3! \times 4! \times 3! \times 2!$$